4 Vector Addition

Practice Problems

4.1 Properties of Vectors pages 64–71

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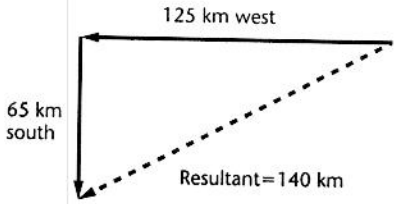
1. A car is driven 125 km due west, then 65 km due south. What is the magnitude of its displacement?



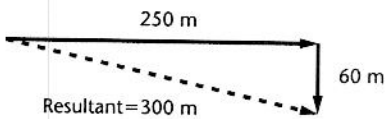
$R^2 = A^2 + B^2$

$R^2 = (65\ \text{km})^2 + (125\ \text{km})^2$

$R^2 = 19\ 850\ \text{km}^2$

$R = 140\ \text{km}$

2. A shopper walks from the door of the mall to her car 250 m down a lane of cars, then turns 90° to the right and walks an additional 60 m. What is the magnitude of the displacement of her car from the mall door?



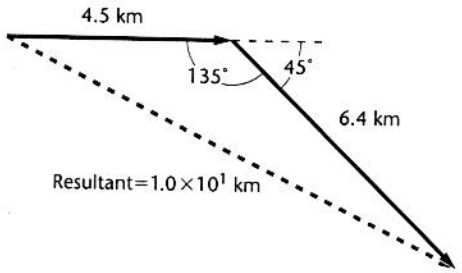
$R^2 = (250\ \text{m})^2 + (60\ \text{m})^2$

$R^2 = 66\ 100\ \text{m}^2$

$R = 260\ \text{m}$, or 300 m to one significant digit

3. A hiker walks 4.5 km in one direction, then makes a 45° turn to the right and walks another 6.4 km. What is the magnitude of her displacement?

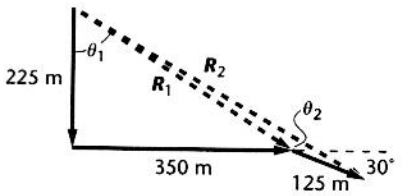
$R^2 = A^2 + B^2 - 2AB\cos\theta$



$R = [(4.5\ \text{km})^2 + (6.4\ \text{km})^2 - (2)(4.5\ \text{km})(6.4\ \text{km})(\cos 135°)]^{1/2}$

$R = 1.0 \times 10^1\ \text{km}$

4. What is the magnitude of your displacement when you follow directions that tell you to walk 225 m in one direction, make a 90° turn to the left and walk 350 m, then make a 30° turn to the right and walk 125 m?



$R_1 = [(225\ \text{m})^2 + (350\ \text{m})^2]^{1/2} = 416\ \text{m}$

$\theta_1 = \tan^{-1}\frac{350\ \text{m}}{225\ \text{m}} = 57.3°$

$\theta_2 = 180 - (60 - 57.3) = 177.3°$

$R_2 = [(416\ \text{m})^2 + (125\ \text{m})^2 - 2(416\ \text{m})(125\ \text{m})(\cos 177.3°)]^{1/2}$

$R_2 = 540\ \text{m}$

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5. A car moving east at 45 km/h turns and travels west at 30 km/h. What are the magnitude and direction of the change in velocity?

5. (continued)

Magnitude of change in velocity

= 45 − (−30) = 75 km/h

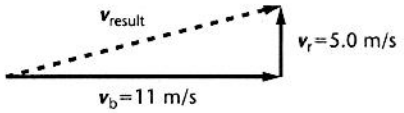
direction of change is from east to west

6. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

+2.0 m/s + 4.0 m/s

= 6.0 m/s relative to street

7. A motorboat heads due east at 11 m/s relative to the water across a river that flows due north at 5.0 m/s. What is the velocity of the motorboat with respect to the shore?



$$v_{result} = [v_b^2 + v_r^2]^{1/2}$$
$$= [(11 \text{ m/s})^2 + (5.0 \text{ m/s})^2]^{1/2}$$
$$= 12 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{5.0 \text{ m/s}}{11 \text{ m/s}} = 24°$$

v_{result} = 12 m/s, 66° east of north

8. A boat is rowed directly upriver at a speed of 2.5 m/s relative to the water. Viewers on the shore find that it is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

2.5 m/s

⟶ boat

2.0 m/s river

⟵

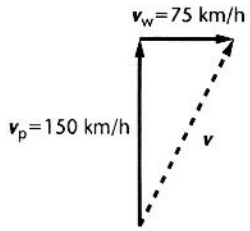
→

0.5 m/s Resultant

2.5 m/s − 0.5 m/s

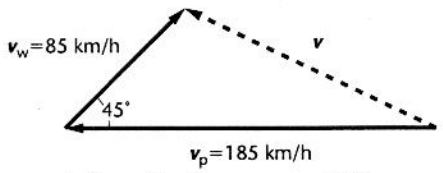
= 2.0 m/s against the boat

9. An airplane flies due north at 150 km/h with respect to the air. There is a wind blowing at 75 km/h to the east relative to the ground. What is the plane's speed with respect to the ground?



$$v = [v_p^2 + v_w^2]^{1/2}$$
$$= [(150 \text{ km/h})^2 + (75 \text{ km/h})^2]^{1/2}$$
$$= 170 \text{ km/h}$$

10. An airplane flies due west at 185 km/h with respect to the air. There is a wind blowing at 85 km/h to the northeast relative to the ground. What is the plane's speed with respect to the ground?

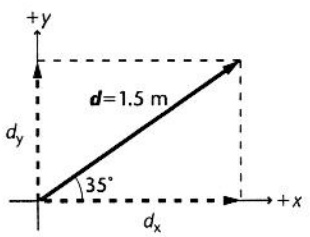


$$v = [v_p^2 + v_w^2 - 2v_p v_w \cos \theta]^{1/2}$$
$$= [(185 \text{ km/h})^2 + (85 \text{ km/h})^2$$
$$- (2)(185 \text{ km/h})(85 \text{ km/h})(\cos 45°)]^{1/2}$$
$$= 140 \text{ km/h}$$

4.2 Components of Vectors pages 72–76

page 74

11. What are the components of a vector of magnitude 1.5 m at an angle of 35° from the positive x-axis?

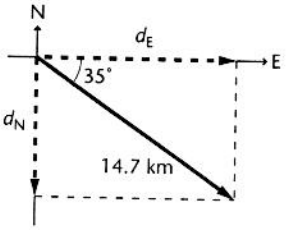


11. (continued)

$d_x = 1.5\text{ m} \cos 35° = 1.2\text{ m}$

$d_y = 1.5\text{ m} \sin 35° = 0.86\text{ m}$

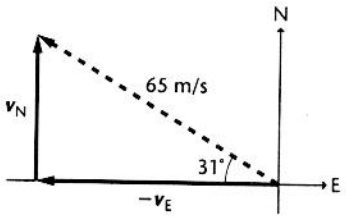
12. A hiker walks 14.7 km at an angle 35° south of east. Find the east and north components of this walk.



$d_E = 14.7\text{ km} \cos 35° = 12.0\text{ km}$

$d_N = -14.7\text{ km} \sin 35° = -8.43\text{ km}$

13. An airplane flies at 65 m/s in the direction 149° counterclockwise from east. What are the east and north components of the plane's velocity?



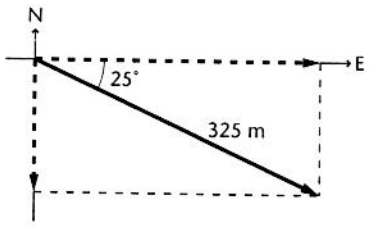
$v_E = -65\text{ m/s} \cos 31° = -55.71\text{ m/s}$

$= -56\text{ m/s}$

$v_N = 65\text{ m/s} \sin 31° = 33.47\text{ m/s}$

$= 33\text{ m/s}$

14. A golf ball, hit from the tee, travels 325 m in a direction 25° south of the east axis. What are the east and north components of its displacement?



$d_E = 325\text{ m} \cos 25° = 295\text{ m}$

$d_N = -325\text{ m} \sin 25° = -137\text{ m}$

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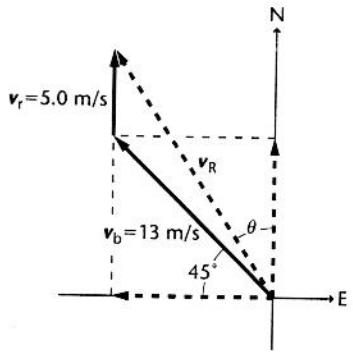
15. A powerboat heads due northwest at 13 m/s with respect to the water across a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat with respect to the shore?



$v_{bW} = (13\text{ m/s}) \cos 45° = 9.2\text{ m/s}$

$v_{bN} = (13\text{ m/s}) \sin 45° = 9.2\text{ m/s}$

$v_{RN} = 5.0\text{ m/s}, v_{rW} = 0.0$

$v_{RW} = 9.2\text{ m/s} + 0.0 = 9.2\text{ m/s}$

$v_{RN} = 9.2\text{ m/s} + 5.0\text{ m/s} = 14.2\text{ m/s}$

$v_R = [(9.2\text{ m/s})^2 + (14.2\text{ m/s})^2]^{1/2}$

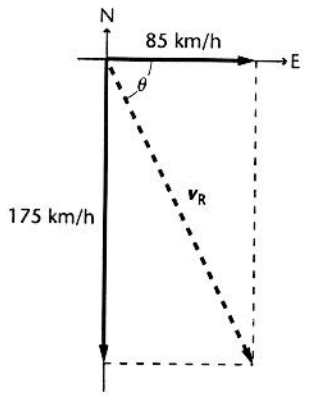
$= 17\text{ m/s}$

$$\theta = \tan^{-1} \frac{9.2\text{ m/s}}{14.2\text{ m/s}} = \tan^{-1} 0.648$$

$= 33°$

$v_R = 17\text{ m/s}, 33°$ west of north

16. An airplane flies due south at 175 km/h with respect to the air. There is a wind blowing at 85 km/h to the east relative to the ground. What are the plane's speed and direction with respect to the ground?



16. **(continued)**

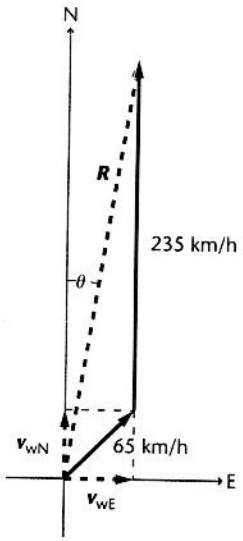
$v_R = [(175 \text{ km/h})^2 + (85 \text{ km/h})^2]^{1/2}$

$= 190 \text{ km/h}$

$\theta = \tan^{-1} \frac{175 \text{ km/h}}{85 \text{ km/h}} = \tan^{-1} 2.06 = 64°$

$v_R = 190 \text{ km/h}, 64°$ south of east

17. An airplane flies due north at 235 km/h with respect to the air. There is a wind blowing at 65 km/h to the northeast with respect to the ground. What are the plane's speed and direction with respect to the ground?



$v_{wN} = 65 \text{ km/h} \sin 45° = 46 \text{ km/h}$

$v_{wE} = 65 \text{ km/h} \cos 45° = 46 \text{ km/h}$

$R_N = 46 \text{ km/h} + 235 \text{ km/h} = 281 \text{ km/h}$

$R_E = 46 \text{ km/h}$

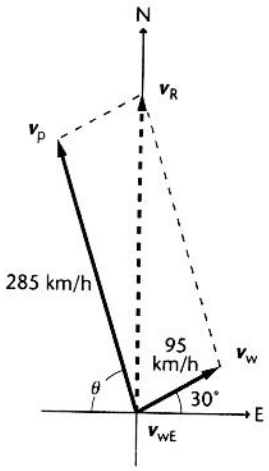
$R = [(281 \text{ km/h})^2 + (46 \text{ km/h})^2]^{1/2}$

$= 280 \text{ km/h}$

$\theta = \tan^{-1} \frac{46 \text{ km/h}}{281 \text{ km/h}}$

$= 9.3°$ east of north

18. An airplane has a speed of 285 km/h with respect to the air. There is a wind blowing at 95 km/h at 30° north of east with respect to Earth. In which direction should the plane head in order to land at an airport due north of its present location? What would be the plane's speed with respect to the ground?



To travel north, the east components must be equal and opposite.

$v_{pE} = v_{wE} = 95 \text{ km/h} \cos 30°$

$= 82 \text{ km/h}$

$\theta = \cos^{-1} \frac{82 \text{ km/h}}{285 \text{ km/h}} = 73°$

$v_{pN} = 285 \text{ km/h} \sin 73° = 273 \text{ km/h}$

$v_{wN} = 95 \text{ km/h} \sin 30° = 47.5 \text{ km/h}$

$v_{RN} = 273 \text{ km/h} + 47.5 \text{ km/h}$

$= 320 \text{ km/h}$

$v_R = 320 \text{ km/h}$ north

Chapter Review Problems

pages 78–79

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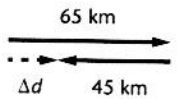
Section 4.1

Level 1

19. A car moves 65 km due east, then 45 km due west. What is its total displacement?

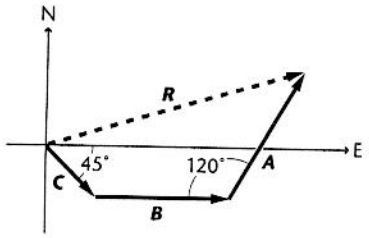


$65 \text{ km} - 45 \text{ km} = 20 \text{ km}$

$\Delta d = 2.0 \times 10^1 \text{ km}$, east

23. (continued)

b. Graphically add the hiker's displacements in the order ***C, B, A.***



c. What can you conclude about the resulting displacements?

You can add vectors in any order. The result is always the same.

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Level 2

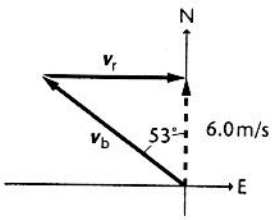
24. A river flows toward the east. Because of your knowledge of physics, you head your boat 53° west of north and have a velocity of 6.0 m/s due north relative to the shore.

a. What is the velocity of the current?

$v_r = (6.0\ \text{m/s})(\tan 53°) = 8.0\ \text{m/s}$

b. What is your speed relative to the water?

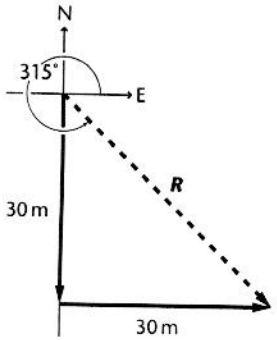
$$v_b = \frac{6.0\ \text{m/s}}{\cos 53°} = 1.0 \times 10^1\ \text{m/s}$$



Section 4.2

Level 1

25. You walk 30 m south and 30 m east. Find the magnitude and direction of the resultant displacement both graphically and algebraically.



$R^2 = A^2 + B^2$

$R = \sqrt{(30\ \text{m})^2 + (30\ \text{m})^2}$

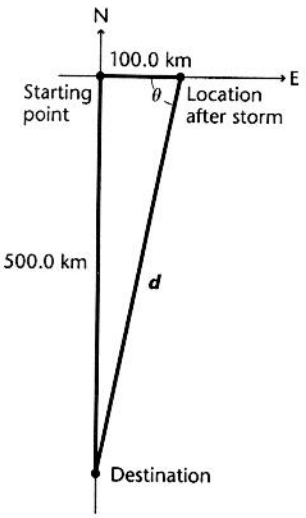
$= \sqrt{1800\ \text{m}^2} = 40\ \text{m}$

$\tan\theta = \dfrac{30\ \text{m}}{30\ \text{m}} = 1$

$\theta = 45°$

$R = 40\ \text{m}, 315°$

26. A ship leaves its home port expecting to travel to a port 500.0 km due south. Before it moves even 1 km, a severe storm blows it 100.0 km due east. How far is the ship from its destination? In what direction must it travel to reach its destination?



$R^2 = A^2 + B^2$

$R = \sqrt{(100.0\ \text{km})^2 + (500.0\ \text{km})^2}$

$= \sqrt{260\ 000\ \text{km}^2}$

$= 509.9\ \text{km}$

$\tan\theta = \dfrac{500.0\ \text{km}}{100.0\ \text{km}} = 5.000$

$\theta = 78.69°$

$d = R = 509.9\ \text{km},$
$78.69°$ **south of west**

20. Graphically find the sum of the following pairs of vectors whose lengths and directions are shown in **Figure 4–12.**

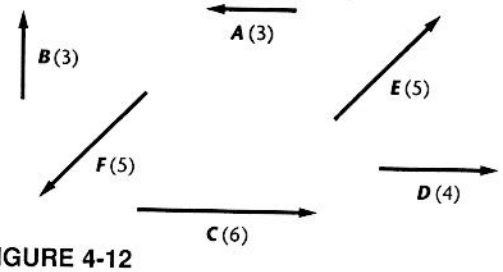
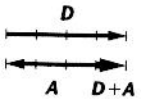
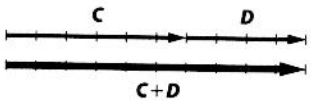


FIGURE 4-12

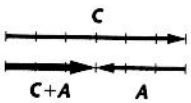
a. ***D*** and ***A***



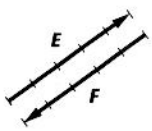
b. ***C*** and ***D***



c. ***C*** and ***A***



d. ***E*** and ***F***



zero

21. An airplane flies at 200.0 km/h with respect to the air. What is the velocity of the plane relative to the ground if it flies with

a. a 50-km/h tailwind?

Tailwind is in the same direction as the airplane

200.0 km/h + 50 km/h = 250 km/h

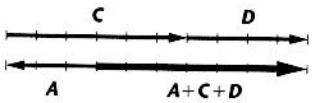
b. a 50-km/h head wind?

Head wind is in the opposite direction of the airplane

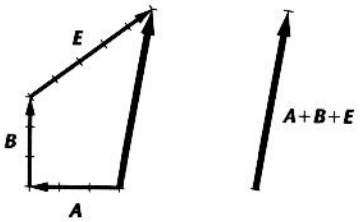
200.0 km/h − 50 km/h = 150 km/h

22. Graphically add the following sets of vectors as shown in **Figure 4–12.**

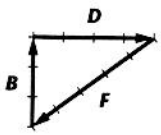
a. ***A, C,*** and ***D***



b. ***A, B,*** and ***E***



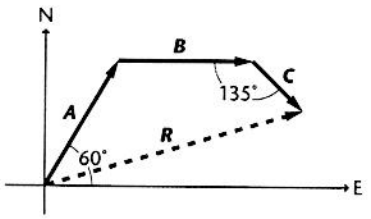
c. ***B, D,*** and ***F***



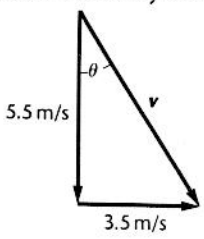
zero

23. Path A is 8.0 km long heading 60.0° north of east. Path B is 7.0 km long in a direction due east. Path C is 4.0 km long heading 315° counterclockwise from east.

a. Graphically add the hiker's displacements in the order ***A, B, C.***



27. A descent vehicle landing on Mars has a vertical velocity toward the surface of Mars of 5.5 m/s. At the same time, it has a horizontal velocity of 3.5 m/s.



a. At what speed does the vehicle move along its descent path?

$R^2 = A^2 + B^2$

$R = \sqrt{(5.5\ \text{m/s})^2 + (3.5\ \text{m/s})^2}$

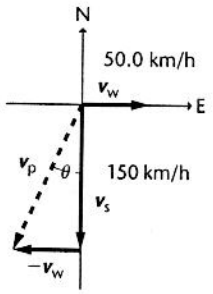
$v = R = 6.5\ \text{m/s}$

b. At what angle with the vertical is this path?

$\tan\theta = \dfrac{3.5\ \text{m/s}}{5.5\ \text{m/s}} = 0.64$

$\theta = 32°$

28. You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 hours. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?



$v_s = \dfrac{d_s}{t} = \dfrac{450\ \text{km}}{3.0\ \text{h}} = 150\ \text{km/h}$

$v_p = [(150\ \text{km/h})^2 + (50.0\ \text{km/h})^2]^{1/2}$

$= 160\ \text{km/h}$

$\theta = \tan^{-1}\dfrac{50.0\ \text{km/h}}{150\ \text{km/h}} = \tan^{-1} 0.33$

$= 18°$ west of south

Level 2

29. A hiker leaves camp and, using a compass, walks 4 km E, then 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and finally 3 km S. At the end of three days, the hiker is lost. By drawing a diagram, compute how far the hiker is from camp and which direction should be taken to get back to camp.

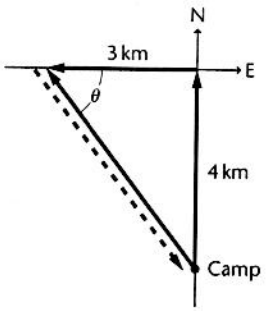
Take north and east to be positive directions.

North: $-6\ \text{km} + 5\ \text{km} + 8\ \text{km} - 3\ \text{km}$

$= 4\ \text{km}$

East: $4\ \text{km} + 3\ \text{km} - 10\ \text{km} = -3\ \text{km}$

The hiker is 4 km north and 3 km west of camp. To return to camp, the hiker must go 3 km east and 4 km south.



$4\ \text{km E} + 3\ \text{km E} + 10\ \text{km W}$

$= 3\ \text{km W}$

$6\ \text{km S} + 5\ \text{km N} + 8\ \text{km N} + 3\ \text{km S}$

$= 4\ \text{km N}$

$R^2 = A^2 + B^2$

$R = \sqrt{(3\ \text{km})^2 + (4\ \text{km})^2} = \sqrt{25\ \text{km}^2}$

$= 5\ \text{km}$

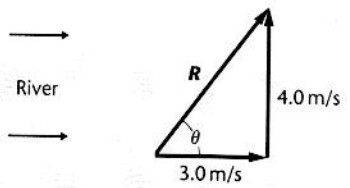
$\tan\theta = \dfrac{4\ \text{km}}{3\ \text{km}} = 1.33$

$\theta = 53°$

$R = 5\ \text{km}, 53°$ south of east

30. You row a boat perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.

a. What is the velocity of your boat relative to the shore?



$R = \sqrt{(3.0\ \text{m/s})^2 + (4.0\ \text{m/s})^2}$

$= 5.0\ \text{m/s}$

$\tan\theta = \frac{4.0}{3.0} = 1.33$

$\theta = 53°$ **with the shore**

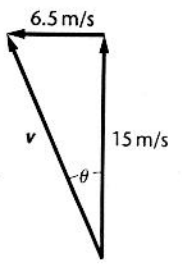
b. What is the component of your velocity parallel to the shore?

3.0 m/s

Perpendicular to it?

4.0 m/s

31. A weather station releases a balloon that rises at a constant 15 m/s relative to the air, but there is a wind blowing at 6.5 m/s toward the west. What are the magnitude and direction of the velocity of the balloon?



$v = \sqrt{(6.5\ \text{m/s})^2 + (15\ \text{m/s})^2} = 16\ \text{m/s}$

$\tan\theta = \frac{6.5\ \text{m/s}}{15\ \text{m/s}} = 0.43$

$\theta = 23°$

$v = 16\ \text{m/s}, 113°$

Critical Thinking Problems

32. An airplane, moving at 375 m/s relative to the ground, fires a missle at a speed of 782 m/s relative to the plane. What is the speed of the shell relative to the ground?

$v_A + v_B = v_R$

$375\ \text{m/s} + 782\ \text{m/s} = 1157\ \text{m/s}$

33. A rocket in outer space that is moving at a speed of 1.25 km/s relative to an observer fires its motor. Hot gases are expelled out the rear at 2.75 km/s relative to the rocket. What is the speed of the gases relative to the observer?

$v_A - v_B = v_R$

$1.25\ \text{km/s} - 2.75\ \text{km/s} = -1.50\ \text{km/s}$